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European Central Counterparty N.V.

Equity Derivatives Margin Model Description (ClearisQ)

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The Clearing Rules of EuroCCP and the margin requirements issued to each Clearing Participant determine the obligations of each Clearing Participant at all times.

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1. Introduction

This document contains a description of EuroCCP's margin model (referred to in this document as the "**ClearisQ**" model) and associated margin add-ons relevant to the clearing of equity index futures and options (collectively referred to as "equity derivatives")[1].

The margin methodologies described in this document are designed to meet the requirements set out under European Market Infrastructure Regulation ("**EMIR**"), and EMIR Regulatory Technical Standards ("**EMIR RTS**").

ClearisQ calculates core initial margin ("**IM**") requirements using a Filtered Historical Simulation ("**FHS**") methodology, based on an Expected Shortfall ("**ES**") tail measure at a confidence level of 99% and a lookback period of 700 business days. The FHS model is supplemented by a Stressed Historical Simulation ("**StressHS**") model in accordance with anti-procyclicality ("**APC**") requirements stipulated under EMIR. An APC-compliant margin is then computed as the weighted sum of the FHS and the StressHS components in line with regulatory requirements.

Where applicable, initial margin add-ons are calculated in addition to core IM to cover exposures related to wrong-way risk, liquidity risk, and large stress loss exposures.

EuroCCP calculates margin requirements on an overnight and intraday basis for every Clearing Participant portfolio containing equity derivatives using the methodologies described in this document.

This document is intended for existing and prospective Clearing Participants, regulators, and internal users.

[1] A margin model description document relevant to cash equity products is available separately.

2. Margin Requirements

2.1 Overview

During the course of each business day, on a continuous basis, EuroCCP calculates a Margin Requirement for each Clearing Participant portfolio containing equity derivative contracts, which is based on five components, and which must be collateralised at all times. Margin Requirements may be collateralised with eligible collateral as listed in the EuroCCP Acceptable Collateral[2] guidelines, subject to the applicable haircut. The margin components used to calculate each Clearing Participant's Margin Requirement are:

- Initial Margin ("IM")
- Futures Variation Margin ("VM")
- Options Premium Margin ("PM")
- Unexpired Net Options Value ("NOV")
- Expired Options NOV

On the morning of each business day, EuroCCP performs the daily cash settlement cycle in each relevant currency (CHF, EUR, and GBP), which facilitates the pass-through of cash settlement amounts pertaining to the previous clearing day, from payers to receivers. The elements comprising the daily cash settlement amounts are:

- Futures daily cash-settled obligations
- Option premium payment obligations
- Option exercise cash-settled obligations

2.2 Futures Variation Margin

Futures Variation Margin represents the intra-day accumulated gain or loss on futures contracts. On day (and time) *t,* it is calculated for a given portfolio as:

$$VM_t = \sum_{ccy} VM_{ccy,t} FX_{ccy,t} \tag{2.1}$$

where $FX_{ccy,t}$ is the foreign exchange spot rate for currency pair *ccy/EUR* (where 1 unit of *ccy* currency buys the quoted amount of EUR currency), and $VM_{ccy,t}$ is the VM in *ccy* currency at *t* calculated as:

$$\begin{aligned} VM_{ccy,t} = \sum_{i_{ccy}} P_{i_{ccy},t} Q_{i_{ccy},t} S_{i_{ccy}} - \sum_{i_{ccy}} P_{i_{ccy},close,t-1} Q_{i_{ccy},close,t-1} S_{i_{ccy}} \\ - \sum_{i_{ccy}} \bar{P}_{i_{ccy},b,t}\, Q_{i_{ccy},b,t} S_{i_{ccy}} + \sum_{i_{ccy}} \bar{P}_{i_{ccy},s,t}\, Q_{i_{ccy},s,t} S_{i_{ccy}} \end{aligned} \tag{2.2}$$

and,

i_{ccy} = futures contract i, denominated in currency ccy

$S_{i_{ccy}}$ = Contract size of options contract i_{ccy}

$P_{i_{ccy},t}$ = Market (or closing) price of futures contract i_{ccy} at *t*

$Q_{i_{ccy},t}$ = Quantity in futures contract i_{ccy} at *t*

[2] https://euroccp.com/document/euroccp-acceptable-collateral/

$P_{i_{ccy},close,t-1}$ = Closing price of futures contract i_{ccy} on day $t-1$

$Q_{i_{ccy},close,t-1}$ = Closing quantity in futures contract i_{ccy} on day $t-1$

$\bar{P}_{i_{ccy},b,t}$ = Weighted average price of futures contract i_{ccy} bought on day t

$Q_{i_{ccy},b,t}$ = Quantity of futures contract i_{ccy} bought on day t

$\bar{P}_{i_{ccy}s,t}$ = Weighted average price of futures contract i_{ccy} sold on day t

$Q_{i_{ccy},s,t}$ = Quantity of futures contract i_{ccy} sold on day t

Positive VM represents a gain to the Clearing Participant, whilst negative VM represents a loss. On an intra-day basis, Futures VM is aggregated across all currencies (in EUR equivalent) and collateralised throughout the clearing day.

Each morning, daily VM cash settled obligations in each currency are settled during the cash settlement cycle. On completion of the daily cash settlement cycle, the previous day's Futures VM resets to zero.

On the expiry date of the futures contract, the final daily Futures VM amount is calculated, and this daily cash settled obligation is settled within the cash settlement cycle on the following business day as usual. The final settlement amount may also be calculated using Eq. 2.2, with the final futures settlement price used in lieu of the closing price. As with the normal daily process, on completion of the daily cash settlement cycle, the previous day's Futures VM resets to zero.

2.3 Options Premium Margin

Option premiums are payable from option buyers to option sellers one business day after trade execution. As such, the settlement of the premium payment obligations occurs as part of the next day's cash settlement cycle.

Options Premium Margin represents the intra-day accumulated premiums payable/receivable on options contracts that have been traded during a clearing day. On day (and time) *t,* it is calculated for a given portfolio as:

$$PM_t = \sum_{ccy} PM_{ccy,t} FX_{ccy,t} \tag{2.3}$$

where $FX_{ccy,t}$ is the foreign exchange spot rate for currency pair *ccy/EUR* (where 1 unit of *ccy* currency buys the quoted amount of EUR currency), and $PM_{ccy,t}$ is the PM in *ccy* currency at **t** calculated as:

$$PM_{ccy,t} = \sum_{i_{ccy}} P_{j,i_{ccy},b,t}\, Q_{j,i_{ccy},b,t} S_{i_{ccy}} - \sum_{i_{ccy}} P_{j,i_{ccy},s,t}\, Q_{j,i_{ccy},s,t} S_{i_{ccy}} \tag{2.4}$$

and,

i_{ccy} = Options contract i, denominated in currency ccy

$S_{i_{ccy}}$ = Contract size of options contract i_{ccy}

$P_{j,i_{ccy},s,t}$ = Price per contract of trade *j* in option i_{ccy}, sold on day **t**

$Q_{j,i_{ccy},s,t}$ = Quantity of trade *j* in option i_m, sold on day **t**

$P_{j,i_{ccy},b,t}$ = Price per contract of trade *j* in option i_m, bought on day **t**

$Q_{j,i_{ccy},b,t}$ = Quantity of trade *j* in option i_m, bought on day **t**

On an intra-day basis, Options PM is aggregated across all currencies (in EUR equivalent) and collateralised throughout the clearing day.

Each morning, Options PM cash settled obligations in each currency are settled during the cash settlement cycle. On completion of the daily cash settlement cycle, the previous day's Options PM resets to zero.

2.4 Unexpired Net Options Value

The Unexpired Net Options Value represents the current value of all unexpired options held within a portfolio. On day (and time) *t,* it is calculated for a given portfolio as:

$$NOV_t = \sum_{ccy} NOV_{ccy,t} FX_{ccy,t} \tag{2.5}$$

where $FX_{ccy,t}$ is the foreign exchange spot rate for currency pair *ccy/EUR* (where 1 unit of *ccy* currency buys the quoted amount of EUR currency), and $NOV_{ccy,t}$ is the NOV in *ccy* currency at *t* calculated as:

$$NOV_{ccy,t} = \sum_{i_{ccy}} P_{i_{ccy},t} Q_{i_{ccy},t} S_{i_{ccy}} \tag{2.6}$$

and,

i_{ccy} = Options contract i, denominated in currency ccy

$S_{i_{ccy}}$ = Contract size of options contract i_{ccy}

$P_{i_{ccy},t}$ = Market (or closing) price of option contract i_{ccy} at *t*

$Q_{i_{ccy},t}$ = Quantity in unexpired option contract i_{ccy} at t (net long positon is a postive value)

The NOV represents the cash amount that would be received or paid upon the liquidation of all options within the portfolio. For this reason, any negative total NOV value must be collateralised to ensure that sufficient resources are available to close out positions in a Clearing Participant default. Positive total NOV values, on the other hand, serve to offset and reduce collateral obligations arising from the other elements within the margin requirement, subject to a floor of zero on the total margin requirement.

2.5 Exercised Net Options Value

Exercised Net Options Value reflects the accrued, but unsettled amounts arising from the exercise of options at time of expiry. On a given day (and time) *t,* it is calculated for a given portfolio as:

$$NOV_{exercised,t} = \sum_{ccy} EXO_{ccy,t} FX_{ccy,t} \tag{2.7}$$

where $FX_{ccy,t}$ is the foreign exchange spot rate for currency pair *ccy/EUR* (where 1 unit of *ccy* currency buys the quoted amount of EUR currency), and $EXO_{ccy,t}$ is the option exercise settlement obligation in *ccy* currency at *t* calculated as:

$$EXO_{ccy,t} = \sum_{c_{ccy}} \max\,(P_{c_{ref}} - K_c, 0) Q_c S_c + \sum_{p_{ccy}} \max\,(K_p - P_{p_{ref}}, 0) Q_p S_p \tag{2.8}$$

and,

c_{ccy} = Call option c denominated in currency ccy

p_{ccy} = Put option p denominated in currency ccy

$P_{c_{ref}}$ = Reference price of underlying index for call option c

K_c = Strike price of call option c

Q_c = Quantity of call option c held at time of expiry

S_c = Contract size of call option c

$P_{p_{ref}}$ = Reference price of underlying index for put option p

K_p = Strike price of put option p

Q_p = Quantity of put option p held at time of expiry

S_p = Contract size of put option p

The reference price for the underlying index is set by the exchange based on the settlement mechanism, as outlined in the contract specifications.

On an intra-day basis, Exercised NOV is aggregated across all currencies (in EUR equivalent) and collateralised throughout the clearing day.

Each morning, Exercised NOV cash settled obligations in each currency are settled during the cash settlement cycle. On completion of the daily cash settlement cycle, the previous day's Exercised NOV resets to zero.

2.6 Aggregation of Margin Requirements

On day (and time) t, the total Margin Requirement that must be collateralised for a given portfolio is calculated based on the above five components, and is floored at zero:

$$Total\ Margin_t = \max\left(IM_t - VM_t + PM_t - NOV_t - NOV_{exercised,t}, 0\right) \tag{2.9}$$

where IM_t is the Initial Margin requirement represented as a positive value.

2.7 Aggregation of Cash-Settled Obligations

On day (and time) t, the total cash amount that is due to be received (or paid) by each Clearing Participant in the next day's cash settlement cycle is calculated in each relevant currency based on the following three components:

$$\begin{aligned} Cash\ Settlement_{ccy} = {} & Futures\ daily\ cash\text{-}settled\ obligations_{ccy} \\ & + Option\ premium\ payment\ obligations_{ccy} \\ & + Option\ exercise\ cash\text{-}settled\ obligations_{ccy} \end{aligned} \tag{2.10}$$

where amounts due to be received by the Clearing Participant are represented as positive values, and vice versa.

The remainder of this document details the calculation of the Initial Margin requirement.

3. Initial Margin methodology overview

3.1 Initial Margin components

The core margin methodology is based on a Filtered Historical Simulation (**FHS**), Expected Shortfall (**ES**)[3] methodology. This model is then supplemented with adjustments designed to improve resilience with respect to procyclicality and netting. Finally, the Initial Margin is supplemented with three margin add-ons designed to address risks arising from wrong-way risk, illiquid positions, and positions that are large relative to the total available resources of the CCP.

3.1.1 Filtered historical simulation

The FHS methodology estimates potential losses based on historical changes (returns) in prices and implied volatilities. The margins calculated using this model represent the potential expected loss for a given portfolio based on EuroCCP's assumed lookback period, liquidation period, and confidence level (see Section 3.2).

The key features of the FHS model are its ability to capture dynamic volatility regime changes and reproduce the joint distribution of returns, without the need to compute the covariance matrix.

The FHS methodology quantifies risk exposure at the portfolio level, i.e. allowing for risk offsets between individual instruments where appropriate. The calculation of margin at the portfolio level is subject to a portfolio margin limit rule in accordance with Article 27(4) of the EMIR RTS.

3.1.2 Anti-procyclicality measure

By construction, the Initial Margin computed using the FHS model adapts to changing market conditions. While this property is desirable from the point of view of efficient capital allocation, it can also lead to procyclical behaviour where margin requirements increase significantly over a short time period in response to market stress.

Article 28 of the EMIR RTS sets out requirements for CCPs to limit the procyclicality of margin requirements to the extent that the soundness and financial security of the CCP is not adversely affected.

To comply with regulatory requirements, EuroCCP applies an anti-procyclicality (APC) measure in the form of a stressed historical simulation (StressHS) margin (ie. by applying the option set out under Article 28(1)(b) of the EMIR RTS).

The StressHS margin component is based on a historical simulation, expected shortfall methodology, in which simulation scenarios are obtained by combining the most recent past returns with a set of defined stressed returns. The margins calculated using this model represent the potential expected loss for a given portfolio based on stressed data observations.

The calculation of StressHS margin at the portfolio level is also subject to a portfolio margin limit rule in accordance with Article 27(4) of the EMIR RTS.

An APC-compliant margin is then computed as the weighted sum of the FHS and the StressHS components (with the stress-based component having a weight of 25%, and the FHS having a weight of 75%, in line with regulatory requirements).

3.1.3 Margin add-ons

In addition to underlying price, and implied volatility market risk, EuroCCP is also exposed to other specific risks which require separate treatments in the form of margin add-ons:

[3] Expected Shortfall is also commonly referred to as Expected Tail Loss (ETL) or Conditional Value-at-Risk (CVaR).

- **Wrong-Way Risk ("WWR") add-on** – reflects the risk due to a Clearing Participant holding long exposure to their own stock, including cases where this exposure arises via equity index futures and/or options on equity index futures. This risk originates from the correlation between the default risk and the credit exposure of the Clearing Participant.
- **Liquidity Risk ("LR") add-on** – reflects the risk due to concentrated and/or illiquid positions in a Clearing Participant portfolio. This add-on captures the potential price impact of liquidating such positions in the event of a default.
- **Large Position ("LP") add-on** – reflects the potential excess stress loss in a Clearing Participant portfolio over and above available financial resources. This add-on captures the excess loss exposure taking into account available mutualised Clearing Fund resources under a Cover-2 standard and the Clearing Participant's individual margin collateral posted.

3.1.4 Total IM requirement

The total IM requirement is the aggregate of the APC-compliant margin plus all the margin add-ons detailed in Section 9.

3.2 Key model assumptions

3.2.1 Confidence level

Article 24 of the EMIR RTS requires that IM shall be calculated for financial instruments that are not OTC derivatives using a minimum confidence level of 99%.

To ensure compliance with the EMIR requirement, EuroCCP calculates margin requirements using an ES risk measure at a confidence level of 99%. For a given confidence level, the ES measure is a more conservative approach than Value-at-Risk ("**VaR**")[4] – therefore, the ES margin result is never less than VaR for a given confidence level.

3.2.2 Lookback window

Article 25 of the EMIR RTS requires that IM shall be calculated based on data covering at least the latest 12 months, and that data used shall capture a full range of market conditions including periods of stress.

EuroCCP satisfies these two requirements by:

- using a lookback window (*N*) of 700 business days; and
- including 50 stress scenarios for computing the StressHS margin component.

3.2.3 Liquidation period (Margin period of risk)

Article 26 of the EMIR RTS requires that the defined time horizon for the liquidation of open positions shall be at least two business days for financial instruments that are not OTC derivatives.

To allow for additional conservativeness, EuroCCP assumes a liquidation period, also referred to as margin period of risk or "**MPOR**" (*h*), of three business days when calculating margin requirements. The computation of the risk over the liquidation period is performed by taking into account possible trending or mean-reverting behaviour of consecutive returns observed over the lookback window.

3.2.4 Portfolio margin limit rule

Article 27(4) of the EMIR RTS requires that, where portfolio margining covers multiple instruments, the amount of margin shall be no greater than 80% of the difference between the sum of the margins for each product

[4] Reference to a VaR-based confidence level is implicit in EMIR.

calculated on an individual basis (ie. gross portfolio margin) and the margin calculated based on a combined estimation of the exposure for the combined portfolio (ie. net portfolio margin).

To ensure compliance with the EMIR requirement, EuroCCP applies a portfolio margin limit rule at all times for the calculation of portfolio margin which is thus computed as the weighted sum of the gross and net portfolio margin (with the net portfolio margin having a weight of 80% in line with regulatory requirements).

3.3 Key model limitations

3.3.1 Procyclicality

Notwithstanding the mitigating impact of the APC adjustment, it is important to recognise that the Initial Margin methodology, as with any risk-based margin calculation, is sensitive and responsive to the prevailing market environment. As such, the methodology will generate higher margin requirements in times of market stress, at precisely the time that market participants may be subjected to increased market risk losses, increased credit risk or even, in some cases, operational risk arising from new, and unanticipated market conditions. While this, in itself, is a limitation, it must be weighed carefully against the benefits of a risk-sensitive measure, namely:

- coverage: the methodology ensures that margin levels cover the expected loss in observed market stress conditions;
- responsiveness: margin levels adapt automatically in an appropriate way to changing market conditions;
- efficiency: margin levels reduce automatically in times of lower volatility, subject to the dampening effect of the APC measure; and
- transparency: margin levels are determined by the data directly, and independently of any meaningful intervention on a day-to-day basis. Losses for individual scenarios may be investigated and explained.

3.3.2 Reliance on historical market data

As with any historical Value-at-Risk based methodology, the initial margin model uses, as its inputs, an extensive set of historical data points for a substantial range of risk factors. As such, it is limited by both the quality and quantity of this data and is sensitive to both errors and omissions. EuroCCP addresses this shortcoming by subjecting historical input data to a rigorous quality assessment, and by performing validation of new market data prior to being used in the margin calculation, with a view to capturing and excluding any erroneous data points. Furthermore, the methodology includes a proxying logic (see Section 8) for addressing cases where data is unavailable, along with an accompanying proxy suppression methodology aimed at safeguarding the robustness of the calculation in the event that a subset of the required inputs are missing for a given day.

3.3.3 Sensitivity to model parameters

The methodology relies on the setting of parameters (see Appendix 1) which affect multiple aspects of the calculation. Although these levels have been determined based on extensive analysis and in line with all applicable regulations and best practice they are also, in a certain sense, arbitrary and could easily be set to different levels, which would produce different initial margin numbers.

3.3.4 Sensitivity to distributional assumptions

Although the Initial Margin methodology generates scenarios using actual observed risk factor returns, and hence avoids making explicit parametric distributional assumptions, it does rely on other non-parametric distributional assumptions e.g.:

- Futures prices and FX rates can be modelled using log-returns, with adjustments for prevailing volatility (Eq. 6.1 and 6.7)

- Implied volatilities may be modelled using absolute returns, without scaling for the volatility-of-volatility (Eq. 5.20)

Assumptions of this kind are unavoidable for any methodology in which future returns are modelled based on observed historical returns; however, they do represent a limitation to the model. If the distribution of future returns for a given risk factor were to deviate dramatically from its historical distribution over a short period of time, portfolios could experience losses in excess of those predicted. This is mitigated by the responsive nature of the model, which will pick up, and incorporate, the most severe of such scenarios and retain them indefinitely through the historical stress scenarios, such that future margin calculations will provide protection against a re-occurrence. Furthermore, the hypothetical stress-testing which drives the sizing of the default fund and, by extension, the large position add-on, does incorporate stresses in excess of those observed historically so that these stresses may also be covered.

3.3.5 Reliance on interpolation and extrapolation methodologies

The model relies on linear interpolation and extrapolation when calculating the futures returns at defined pivot points and when calculating instrument P&Ls from these pivot points. It is also used when generating implied volatilities at specified pivot points, and interpolating from these points to the points required to generate P&L for Clearing Participant portfolios. This latter step also employs spline interpolation (see Section 5.4.2). While this methodology is theoretically robust and is in line with best practice, it is also true to say that each time a form of interpolation or extrapolation is used, it introduces a small element of imprecision to the calculation which can prove difficult to accurately quantify. In addition, the choice of interpolation methodology is somewhat arbitrary, and another method might be equally defensible. The methodology aims to address this by choosing, in all cases, to use the least complex method of interpolation and extrapolation, subject to the requirements of the specific circumstances.

3.3.6 Reliance on calibration of implied volatility surfaces

As outlined in Section 5.2, the methodology uses the mixture of lognormals methodology to construct arbitrage-free implied volatility surfaces. This is one of many methods that could have been used and relies on a number of additional assumptions regarding the behaviour of implied volatilities. Although this is a commonly-used methodology with well-documented advantages, it also has a number of limitations. The choice of calibration model is important because, if the model does not provide a sufficiently close fit to the observed volatility surface by, for example, understating the slope, curvature or time-dependence, then it is possible that the model will understate the variability of the implied volatility pivot point that are used within the construction of the options P&L scenarios. EuroCCP mitigates this risk by conducting back-testing, which compares actual realised returns in options prices vs the returns distribution. In this way, any potential shortcoming of the model can be identified.

4. Futures scenarios

This section describes the computation of P&L scenarios for futures to be fed into the margin calculation.

In order to capture the dynamics of futures prices, it is necessary to model the term structure of the underlying index. The main drivers of the term structure are interest rates and expected dividends. Instead of explicitly modelling these two quantities, the approach adopted is to model the futures returns directly at a set of static times-to-maturities ("pivot points"). This approach requires the following steps:

- derivation of the futures price from available option prices through put-call parity;
- derivation of the futures returns for the available maturity dates; and
- interpolation of the futures returns at the required set of maturity points.

As outlined in Section 3.2, in order to compute Initial Margins for a given product, the FHS methodology described in Section 6 requires *N* scenarios for this product over a horizon *h* (the holding period or *MPOR*). These scenarios are built from past data within a time period immediately preceding the calculation date t_0 and referred to as the lookback window.

EuroCCP adopts a lookback window of *N* = 700 business days. More specifically, the methodology uses daily returns within the lookback period and $N^* = N + h - 1$ denotes the total number of such daily returns needed to compute *N* scenarios with MPOR *h* (see Section 6.3).

4.1 Synthetic futures prices

4.1.1 Synthetic futures motivation

Synthetic futures $F_{t,T}^i$ at time *t* for maturity date *T* and underlying *i* can be generated by combining a call and a put of maturity *T* and of same strike *K*, according to put-call parity. This property is used to compute the futures' prices with the following benefits:

- accuracy: the options market is often more liquid than the futures one (especially for equity underliers).
- consistency: some quantities of interest (e.g. margins for the options) depend both on futures and options prices (e.g. through implied volatilities) so it is desirable to have data coming from the least possible number of sources in order to avoid discrepancies.

For the current purpose, only the options where both calls and puts are available for a given maturity/strike couple (*T*,*K*) are considered. Because in-the-money ("**ITM**") options are usually less liquid, it is desirable to select pairs of call/put options with the strike close to the futures price (which is, however, yet to be determined). As a fallback heuristic, the median synthetic future price derived from the 3 couples that have the strike closer to the underlying spot price S^i is used (implicitly assuming a low value for the discount and dividend term). As a fallback, if no pairs of call/put for the same strike and maturity are available, the quoted futures price will be directly used.

4.1.2 Synthetic futures price derivation

Given the maturity-dependent, risk-free interest rate *r*, the price of a synthetic future of maturity *T* may be deduced for European options using put-call parity:

$$F_{t,T}^i = K + \left(C_{t,T,K}^i - P_{t,T,K}^i\right)e^{r(T-t)} \tag{4.1}$$

with $C_{t,T,K}^i$ and $P_{t,T,K}^i$ call and put option prices at time $t \in [1, N^* + 1]$ with maturity *T* and strike *K*.

4.2 Synthetic futures returns

4.2.1 Synthetic futures returns derivation

The futures returns are calculated from the (synthetic) future prices, as logarithmic returns. Compared to arithmetic ones, logarithmic returns have the advantage of being additive, and this property is used afterwards for both the MPOR scaling, and the FX adjustment.

Given the prices $F_{t,T}$ for futures contracts of maturity T on date t (where the underlying dependency has been dropped for ease of notation), the daily logarithmic returns are calculated as:

$$p_{t,\tau} = ln\left(\frac{F_{t,T}}{F_{t-1,T}}\right) \quad \forall t \in [1, N^*] \tag{4.2}$$

where $\tau = T - t$ is the time-to-maturity, which is used to index the returns in the term-structure dimension. Note that returns are computed over $F_{t,T}$ and $F_{t-1,T}$ which are the prices for the same contract maturity on two consecutive days. For every time t, $S(t) = \{\tau_1, , , \tau_s\}$ represents the ensemble of all available interpolated time-to-maturity points, assumed to be in ascending order without loss of generality. This ensemble depends on the available option maturities for a given date.

4.3 Futures return pivot points

4.3.1 Motivation for pivot points

As stated above, the set of observable time-to-maturity points, collected in *S(t)*, varies from one date to another. For the purposes of this methodology, it is necessary to use these points to interpolate a set of returns at a fixed set of defined points, for two reasons:

- to have complete time series with constant statistical properties over the lookback window $t = 1,\dots,N^*$ for every risk factor; and
- to allow for efficient reconstruction of futures returns at any required maturity.

To ensure that the scenarios are free from any discontinuities that could arise from corporate actions, such as discrete dividends, interpolation is performed in the futures returns space rather than the futures price space.

4.3.2 Definition of pivot points

A set of pivot time-to-maturities $P = \tau_1, \dots \tau_{N_\tau}$ is defined to model the term structure of each index. As best practice, the selection of pivot points:

- includes the spot (zero-time-to-maturity) so that $\tau_1 = 0$;
- have more pivot times-to-maturity at shorter maturities, to reflect the greater number of contracts, and greater liquidity at shorter maturities and
- includes the farthest possible maturity, so that τ_{N_τ} = 24 months.

With these in mind, the following points were selected: {0, 1m, 2m, 3m, 6m, 9m, 12m, 18m, and 24m}.

4.3.3 Interpolation from available data points to defined pivot points

A linear interpolation is used:

$$p_{t,\tau_j} = \frac{\tau_j - \tau_a}{\tau_b - \tau_a} p_{t,\tau_a} + \frac{\tau_b - \tau_j}{\tau_b - \tau_a} p_{t,\tau_b} \quad \forall j \in [1, \dots, N_\tau] \tag{4.3}$$

where $\tau_a < \tau_j$ and $\tau_j < \tau_b$, representing the two time-to-maturities that are available in *S(t)* and closest to τ_j. Because the spot is included, it is always possible to find a lower bound to interpolate from. For longer-dated maturities, however, it may be necessary to extrapolate from the furthest available data point. In the absence of a suitable upper pivot, flat extrapolation is used which is supported by the observation that long-dated futures typically exhibit very strong correlations:

$$p_{t,\tau_j} = p_{t,\tau_S} \quad \text{if} \quad \tau_j > \tau_s = \max\big(S(t)\big) \tag{4.4}$$

Utilising both interpolation and, where necessary, extrapolation guarantees that these "futures return pivot points" can always be defined. A collection of N_τ futures returns is then stored at each date needed to build the lookback window, corresponding to $N^* \times N_\tau$ returns. For computational convenience, the returns over the pivot times-to-maturity can be collected into a matrix where row *k* contains the returns on day $t - k + 1$ and column *j* contains the returns for the *j*th pivot time-to-maturity:

$$p = \begin{pmatrix} p_{t,\tau_1} & p_{t,\tau_2} & \cdots & p_{t,\tau_{N_\tau}} \\ p_{t-1,\tau_1} & p_{t,\tau_2} & \cdots & p_{t,\tau_{N_\tau}} \\ \cdots & \cdots & \ddots & \cdots \\ p_{t-N^*+1,\tau_1} & p_{t-N^*+1,\tau_2} & \cdots & p_{t,\tau_{N_\tau}} \end{pmatrix} \tag{4.5}$$

Using these 1-day, unscaled returns (*p*) it is possible to construct a matrix of *MPOR*-scaled hypothetical returns ($\hat{r}$) representing the modelled distribution of scaled future returns at each pivot point over the 3-day *MPOR* horizon. Sections 6.2 and 6.3 provide further details on this transformation.

$$p \to \hat{r} = \begin{pmatrix} \hat{r}_{t,\tau_1} & \hat{r}_{t,\tau_2} & \cdots & \hat{r}_{t,\tau_{N_\tau}} \\ \hat{r}_{t-1,\tau_1} & \hat{r}_{t,\tau_2} & \cdots & \hat{r}_{t,\tau_{N_\tau}} \\ \cdots & \cdots & \ddots & \cdots \\ \hat{r}_{t-N^*+1,\tau_1} & \hat{r}_{t-N^*+1,\tau_2} & \cdots & \hat{r}_{t,\tau_{N_\tau}} \end{pmatrix} \tag{4.6}$$

4.4 Interpolation from pivot points to required maturities

For every underlying index, a collection of *MPOR*-scaled returns over the lookback window has been computed at pivot maturities $\tau_1, \dots, \tau_{N_\tau}$ as detailed above. It is therefore possible to re-interpolate the returns at the required time-to-maturity τ. The same linear interpolation as in Section 4.3 is employed:

$$\hat{r}_{t,\tau} = \frac{\tau - \tau_a}{\tau_{j_b} - \tau_{j_a}} \hat{r}_{t,\tau_{j_a}} + \frac{\tau_b - \tau}{\tau_{j_b} - \tau_{j_a}} \hat{r}_{t,\tau_{j_b}} \quad \forall t \in [1, \dots, N^*] \tag{4.7}$$

where $\tau_{j_a} < \tau < \tau_{j_b}$ is the closest bracketing using time-to-maturities available in the pivots *P(t)*. Extrapolation is not required for this step, because the longest-dated pivot point (24m) represents the longest possible maturity for a traded market futures contract.

4.5 Futures P&L scenarios

Given $F_{t_0}(\tau)$, the price on today's date t_0 of a futures contract at time-to-maturity τ, a set of price scenarios and a set of P&L scenarios is computed. The set of scenarios over horizon δ_h (spanning *h* business days) for the futures prices (levels), denoted by $\widehat{\cdot}^{\,t}$, reads:

$$\hat{F}^t_{t_{0+h}}(\tau - \delta_h) = F_{t_0}(\tau)\, e^{\hat{r}_{t_0,\tau}} \quad \forall t \in [1, N] \tag{4.8}$$

The set of P&L scenarios (variations), denoted by $\widehat{\Delta^h \cdot}^{\,t}$, reads:

$$\widehat{\Delta^h F}^t_{t_0}(\tau) = \hat{F}^t_{t_0+h}(\tau - \delta_h) - F_{t_0}(\tau) \quad \forall t \in [1, N] \tag{4.9}$$

In case of mismatch between the currency of the index and the Clearing Participant margining currency, Eq. 4.9 is replaced with the variation below, which incorporates the corresponding FX scenario:

$$\widehat{\Delta^h F}^t_{t_0}(\tau) = \widehat{FX}^t_{t_0+h}\left(\hat{F}^t_{t_0+h}(\tau - \delta_h) - F_{t_0}(\tau)\right) \qquad \forall t \in [1, N] \tag{4.10}$$

As with the futures return scenarios, FX scenarios are generated using the FHS approach which is detailed in Section 6.4. The calculation of returns in Eq. 4.10 reflects the fact that the FX exposure only affects VM, and not the full notional value of the futures contract.

Once these variations have been calculated, the position-level P&L scenarios may be calculated by multiplying by the position size and quantity:

$$X^t_{i_\tau} = \widehat{\Delta^h F}^t_{t_0}(\tau) Q_{\mathrm{i},} S_{\mathrm{i}} \qquad \forall t \in [1, N] \tag{4.11}$$

where:

$X^t_{i_\tau}$ is the position-level P&L scenario (t) for futures contract *i* with time-to-maturity τ

$\widehat{\Delta^h F}^t_{t_0}(\tau)$ is the single-contract-level P&L scenario (t) for futures contract *i* with time-to-maturity τ

Q_i is the quantity of future *i*, in a given portfolio

S_i is the contract size of option *i,*

The position P&Ls can then be aggregated across all positions within the portfolio, along with those of the options positions, (see Section 5.4.3) to calculate the overall portfolio-level P&L scenarios:

$$X_t = \sum_i X^t_i \qquad \forall t \in [1, N] \tag{4.12}$$

It is these portfolio-level P&L scenarios that are carried forward to the next stage of the FHS calculation (see Section 6.6).

5. Options scenarios

As is the case for futures, the FHS calculation for options requires N=700 portfolio P&L scenarios. More specifically, the calculation requires $N^* = N + h - 1$ daily P&L returns per instrument, where h denotes the *MPOR*. This section describes the computation of these scenarios, which will be used within the margin calculation described in Section 6.6.

Futures prices and implied volatilities are key risk-factors required in the options pricing formula. Futures price scenarios have been covered in the previous section. This section addresses implied volatility and option price scenarios specifically, focusing on vanilla European options.

5.1 Inversion of implied volatility

Modelling non-linear products requires computing the implied volatilities out of available options. The process of deriving implied volatilities from listed option market data requires the following inputs:

- risk-free interest rate curve: inferred from vanilla interest rate products (sourced directly from market data vendors);
- futures term structure: derived from listed option prices using the put-call parity formula (see Section 4.1); and
- vanilla option prices: used to invert the option pricing formula to derive the implied volatilities.

5.1.1 Pricing formula inversion

The derivation of implied volatility is based on the inversion of an option pricing formula. As the pricing formula takes as input the underlying volatility, one looks for the volatility that matches the observed option price, resulting in the implicit definition of the so-called implied volatility:

$$\sigma_{impl} \quad \text{solves} \quad \left|P\left(\sigma_{impl}\right) - P^{Market}\right| = 0 \tag{5.1}$$

Even in the simplest case of European options, there is no analytical solution for this inversion problem and the solution is computed numerically. As the option price is monotone in the volatility parameter, the unique solution is usually reached in a few iterations. If the optimizer does not converge to a solution, the data point is disregarded; this can happen because the option price violates some arbitrage constraints (e.g. price lower than the intrinsic value).

5.1.2 European options pricing formula

In the case of European option, the implied volatility can be derived by inverting Black's formula, which gives the price of such option as a function of volatility and futures price. It reads, in its general form for both calls and puts:

$$P\left(F_{t,\tau}, \sigma\right) = e^{r\tau}\left(\phi F_{t,\tau} \mathrm{N}(\phi d_1\right) - \phi K \mathrm{N}(\phi d_2)) \tag{5.2}$$

$$d_1 = \frac{ln\left(\frac{F_{t,\tau}}{K}\right) + \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}} \tag{5.3}$$

$$d_2 = d_1 - \sigma\sqrt{\tau} \tag{5.4}$$

where ϕ = 1 for a call option and ϕ = 1 for a put option, $F_{t,\tau}$ is the futures price at time *t* for time to maturity τ, *r* is the τ-dependent risk-free rate of the underlying currency, *K* is the strike and σ is the implied volatility.

5.2 Fitting a volatility surface: mixture of lognormal densities

5.2.1 Arbitrage-free implied volatility surfaces

The subsequent computations require implied volatilities at pairs (τ,*m*) of time-to-maturity and moneyness[5] points, that do not correspond to directly available, i.e. quoted, options hence requiring interpolation. Rather than performing pointwise interpolations, it is deemed more accurate and efficient to calibrate a full volatility surface in an integrated manner. One of the main drivers is to ensure that the implied volatility surface is arbitrage-free, i.e. that the prices associated to any pairs of points (τ_1,m_1) and (τ_2,m_2) of the volatility surface do not yield riskless profit.

For calibration of implied volatility surfaces, the mixture of lognormals model[6] is adopted as it guarantees both flexibility and arbitrage-free surfaces. Alternative models do not perform equally well as the selected model; for instance, spline-based models are more flexible but constraints to ensure arbitrage free surfaces are difficult to introduce; on the other hand, models such as the Stochastic Volatility Inspired (SVI) one, are less flexible, especially after accounting for the constraints required to prevent arbitrage violations in the time-to-maturity dimension.

5.2.2 Mixture of lognormals pricing formula

Following Brigo and Mercurio (2003), a set of n so-called “instrumental" processes are considered, with their stochastic processes modelled as:

$$dS_t(t) = \mu_i(t)S_i(t)dt + \sigma_i(t)S_i(t)dW_t, \quad S_i(0) = S_0 \tag{5.5}$$

where,

> W_t is the standard Wiener process,
>
> μ_i are the drifts, a deterministic function of time defined on [0; T^*],
>
> σ_i are the volatilities, a set of deterministic functions satisfying a given Lipschitz condition to avoid finite-time explosion.

The density of S_i at time *t* is thus given by:

$$p_t^i(S) = \frac{1}{SV_i(t)\sqrt{2\pi}} exp\left[-\frac{1}{2V_i^2(t)}\left(ln\frac{S}{S_0} - M_i(t) + \frac{1}{2}V_i^2(t)\right)^2\right] \tag{5.6}$$

$$M_i(t) = \int_0^t \mu_i(u)du, \tag{5.7}$$

$$V_i(t) = \sqrt{\int_0^t \sigma_i^2(u)du}, \tag{5.8}$$

Assuming that the marginal distribution of *S(t)* is given by a mixture of the above densities,

$$p_t(y) := \frac{d}{dy}Pr\{S(t) \leq y\} = \sum_{i=1}^{n} \lambda_i p_t^i(y) \tag{5.9}$$

with positive mixture weights such that $\sum_{i=1}^{n} \lambda_i$. It is proven in Brigo and Mercurio (2003) that S_t follows the following stochastic differential equation, in the spirit of a local volatility model:

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[5] Defined as the natural logarithm of the ratio of the option strike and the underlying futures.

[6] D. Brigo, F. Mercurio, and G. Sartorelli. Alternative asset-price dynamics and volatility smile. *Quantitative Finance*, 3(3):173-183, 2003.

$$dS(t) = \mu(t)S(t)dt + \Psi\big(t, S(t)\big)S(t)dW_t, \quad S(o) = S_0 \tag{5.10}$$

with,

$$\sum_{i=1}^{N} \lambda_i e^{M_i(t)} = e^{\int_0^t \mu(u)du}, \quad \forall t > 0 \tag{5.11}$$

in order to keep the expectation of the mixture process to $E\big(S(t)\big) = S_0 e^{\int_0^t \mu(u)du}$ at all times, and with:

$$\Psi(t,y)^2 = \frac{\sum_{i=1}^{N} \lambda_i \sigma_i(t)^2 p_t^i(y)}{\sum_{i=1}^{N} \lambda_i p_t^i(y)} + \frac{2S_0 \sum_{i=1}^{N} \lambda_i\big(\mu_i(t) - \mu(t)\big)e^{M_i(t)}\Phi\left(\frac{ln\left(\frac{S_0}{y}\right) + M_i(t) + \frac{1}{2}V_i^2(t)}{\eta_i\sqrt{t}}\right)}{y^2 \sum_{i=1}^{N} \lambda_i p_t^i(y)} \tag{5.12}$$

The pricing of options, under dynamics 5.10, is quite straightforward: the European call option price $C(\tau, K)$, at time t = 0, is given by a linear combination of Black-Scholes prices, namely:

$$C(\tau, K) = e^{r\tau} \sum_{i=1}^{n} \left[S_0 e^{M_i(t)} \Phi\left(\frac{ln\left(\frac{S_0}{K}\right) + M_i(T) + \frac{1}{2}\eta_i^2 T}{\eta_i\sqrt{T}}\right) - K\Phi\left(\frac{\ln\left(\frac{S_0}{K}\right) + M_i(T) - \frac{1}{2}\eta_i^2 T}{\eta_i\sqrt{T}}\right)\right] \tag{5.13}$$

where $\eta_i = V_i(T)/\sqrt{T}$.

The model nicely leads to smiles in the implied volatility structure. Moreover, the different drift rates in the S_i-dynamics allows for reproduction of steeper and more skewed curves than in the common drift case (i.e. $\mu_i(t) = \mu(t)\ \forall_i$), with minimums that can be shifted far away from the at-the-money (“**ATM”**) level.

5.2.3 Fitting algorithm for the mixture of lognormals

It is assumed that the drifts and volatilities of the “instrumental" processes are piecewise constant functions over *m* intervals defined by the observed option time-to-maturities, therefore resulting in the following set of parameters:

$$(\lambda_i)_{1\le i\le n}, \big(\bar{\mu}_{i,j}, \eta_{i,j}\big)_{1\le i\le n, 1\le j\le m} \tag{5.14}$$

where $\bar{\mu}_{i,j} = M_{i,j}/\tau_j$ denotes the average drift up to τ_j for the i-th “instrumental" process.

The operation of fitting the parameters of a n-lognormal mixture model can be represented as follows:

$$\mathcal{F}: \quad \mathbb{R}_{+*}^{N_0} \rightarrow \mathbb{R}^{n(1+2m)}$$

$$(P_0)_{1\le 0\le N_0} \quad \mapsto (\lambda_i)_{1\le i\le n}, \big(\bar{\mu}_{i,j}, \eta_{i,j}\big)_{1\le i\le n, 1\le j\le m} \tag{5.15}$$

where P_0 denotes the observed market price for the *o*-th option among N_0 available out-of-the-money options.

Fitting consists in practice of an optimization algorithm where the objective function to minimize is:

$$\mathcal{L}\left((\lambda_i)_{1\le i\le n},\left(\bar{\mu}_{i,j},\eta_{i,j}\right)_{1\le i\le n,1\le j\le m}\right)=\sum_{0=1,\dots,N_0}\left(\sigma_0^{model}-\sigma_o\right)^2 \tag{5.16}$$

with σ^{model} depending on the model parameters.

The following requirements are included as optimization constraints:

- the average drift of the components should match the expected drift, implicit in the futures price, of the underlying at options maturity (see Eq. 5.11); and
- the total volatilities $V_i(t)$ increases with time-to-maturity.

To avoid recomputing the implied volatilities σ_0^{model} for every data point *o* from the model parametrization at every iteration, the following approximation for the objective function is employed:

$$\mathcal{L}\left((\lambda_i)_{1\le i\le n},\left(\bar{\mu}_{i,j},\eta_{i,j}\right)_{1\le i\le n,1\le j\le m}\right)\approx\sum_{0=1,\dots,N_0}\left(\frac{P_o^{model}-P_o}{v_o}\right)^2 \tag{5.17}$$

where v_o are the vegas of the quoted options, which can be extracted once for all. The vegas actually depend on the implied volatilities σ_o so it is necessary to extract the latter from the prices with the inversion algorithm 5.1 at initialization time. At each iteration of the optimizer, the pricing formula 5.13 is then directly used for computing the P_0^{model}.

In order to obtain stable and smooth optimization results the following enhancements are applied:

- the previous day's parameters for each maturity are used as initial values for the optimization algorithm; and
- a smoothing penalization factor is included in the objective function:

$$\begin{aligned}\mathcal{L}&\left((\lambda_i)_{1\le i\le n},\left(\bar{\mu}_{i,j},\eta_{i,j}\right)_{1\le i\le n,1\le j\le m}\right)\\ &\approx\sum_{0=1,\dots,N_0}\left(\frac{P_o^{model}}{v_o}\right)^2\\ &+\iota\left(\sum_{1\le i\le n,1\le j\le m-1}\left(\mu_{i,j+1}-\mu_{i,j}\right)^2+\sum_{1\le i\le n,1\le j\le m-1}\left(\eta_{i,j+1}-\eta_{i,j}\right)^2\right)\end{aligned} \tag{5.18}$$

where ι is a penalization weight. Note that the fitting errors in the implied volatility space and the penalization terms are of the same order of magnitude, therefore ι = 0:05 is selected as a reasonable parameter to ensure that the fitting part has higher weight than the penalization one.

5.3 Implied volatility pivot points

Consider an option of price $P(\tau,m)$ on today's date t_0, with time-to-maturity τ and moneyness *m*. In order to compute a risk measure, the margin model requires a distribution of possible outcomes from a set of P&L scenarios. For this option, the latter are computed from scenarios of:

- the price of the futures contract with option time-to-maturity τ; and
- the implied volatility at option time-to-maturity τ and moneyness m.

In order to derive implied volatilities at any combination of (τ;m), a set of pivot points is defined on a grid of time-to-maturities $\left(\tau_j\right)_{1\le j\le N_\tau}$ and moneyness $(m_k)_{1\le k\le N_m}$. These pivot points are responsible for driving the dynamics of the entire implied volatility surface and interpolation is used to derive an implied volatility level at any arbitrary point of the surface.

5.3.1 Implied volatility levels at pivot points

The initial inputs for the option scenarios are quoted prices $\left(\left(P_{t,o}\right)_{1\leq o\leq N_{o_t}}\right)_{1\leq t\leq N*}$ for each date t in the lookback window and for whatever available time-to-maturity and moneyness of a generic option o.

On each date t, a volatility surface can be fitted using the algorithm outlined in Section 5.2.3, and implied volatilities can then be derived at each pivot time-to-maturity and moneyness:

$$\left(P_{t,o}\right)_{1\leq o\leq N_{o_t}} \rightarrow (\lambda_i)_{1\leq i\leq n}, \left(\bar{\mu}_{i,j}, \eta_{i,j}\right)_{1\leq i\leq n, 1\leq j\leq m} \rightarrow \left(\sigma_{t,j,k}\right)_{1\leq j\leq N_\tau, 1\leq k\leq N_m} \quad \forall t \tag{5.19}$$

As a result of this process, a time-series of implied volatilities at each implied volatility pivot point is derived.

5.3.2 Implied volatility returns at pivot points

Having fitted the implied volatility surface on all dates $t \in [1, N]$, it is then possible to compute returns at each pivot points as:

$$\Delta\sigma_{t,\tau_j,m_k} = \sigma_{t+1,\tau_j,m_k} - \sigma_{t,\tau_j,m_k} \quad \forall j,k \tag{5.20}$$

for all dates t. Note that the implied volatility returns are calculated as the arithmetic difference between each pair of volatilities, as opposed to the case of futures, described in Eq. 4.2, where logarithmic returns are used.

All implied volatility returns can be collected in matrix form as below, with the elements of each row containing returns at every combination of time to maturity (τ) and moneyness (m).

$$\Delta\sigma = \begin{pmatrix} \Delta\sigma_{t,\tau_1,m_1} & \Delta\sigma_{t,\tau_1,m_2} & \cdots & \Delta\sigma_{t,N_\tau,N_m} \\ \Delta\sigma_{t-1,\tau_1,m_1} & \Delta\sigma_{t-1,\tau_1,m_2} & \cdots & \Delta\sigma_{t-1,N_\tau,N_m} \\ \cdots & \cdots & \ddots & \cdots \\ \Delta\sigma_{t-N^*+1,\tau_1,m_1} & \Delta\sigma_{t-N^*+1,\tau_1,m_2} & \cdots & \Delta\sigma_{t-N^*+1,N_\tau,N_m} \end{pmatrix} \tag{5.21}$$

5.4 Options P&L scenarios

5.4.1 Implied volatility scenarios at pivot points

Using these historical returns, a set of *MPOR*-scaled volatility scenario returns is calculated by summing the historical returns over the t+h-1 consecutive observations.

$$\widehat{\Delta\sigma}_{t,j,k} = \sum_{\theta=t}^{t+h-1} \Delta\sigma_{\theta,j,k} \tag{5.22}$$

This is performed over all time, moneyness and maturity points to produce a comprehensive set of scenario returns at all pivot points for all *N* historical scenario dates.

$$\Delta\sigma \rightarrow \widehat{\Delta\sigma} = \begin{pmatrix} \Delta\hat{\sigma}_{t,\tau_1,m_1} & \Delta\hat{\sigma}_{t,\tau_1,m_2} & \cdots & \Delta\hat{\sigma}_{t,N_\tau,N_m} \\ \Delta\hat{\sigma}_{t-1,\tau_1,m_1} & \Delta\hat{\sigma}_{t-1,\tau_1,m_2} & \cdots & \Delta\hat{\sigma}_{t-1,N_\tau,N_m} \\ \cdots & \cdots & \ddots & \cdots \\ \Delta\hat{\sigma}_{t-N^*+1,\tau_1,m_1} & \Delta\hat{\sigma}_{t-N^*+1,\tau_1,m_2} & \cdots & \Delta\hat{\sigma}_{t-N^*+1,N_\tau,N_m} \end{pmatrix} \tag{5.23}$$

Using these *MPOR*-scaled volatility returns, a set of *N* volatility surface scenarios denoted by $\hat{\sigma}$ is computed by adding the current implied volatility to the corresponding volatility return at each maturity and moneyness point for all dates $t \in [1, N]$ in the lookback window.

$$\hat{\sigma}^{t}_{t_0+h,\tau_j,m_k} = \sigma_{t_0,\tau_j,m_k} + \widehat{\Delta\sigma}^{t}_{\theta,\tau_j,m_k} \qquad \forall j,k,t \tag{5.24}$$

The superscript *t* recalls that the scenario is built from historical data at date index *t*.

5.4.2 Implied volatility scenarios at arbitrary point

To recover the scenarios at the target time-to-maturity τ and moneyness *m*:

$$\left(\hat{\sigma}^{t}_{t_0+h,j,k}\right)_{j,k} \rightarrow \hat{\sigma}^{t}_{t_0+h}(\tau, m) \tag{5.25}$$

a two-step process is performed as followed, (dropping the time and scenario notations):

1. a pair of implied volatility scenarios $\left(\hat{\sigma}_{j_a}(m), \hat{\sigma}_{j_b}(m)\right)$ are built at the target moneyness m using spline interpolation (with linear extrapolation) for the two closest time-to-maturity slices τ_{j_a} and τ_{j_b} at which pivot points are defined,
2. the implied volatility scenario at data time-to-maturity τ is derived from these two points using a linear interpolation based on total variance $V = \sigma^2\tau$:

$$\hat{\sigma}^2(\tau, m)\tau = \frac{\tau - \tau_{j_a}}{\tau_{j_b} - \tau_{j_a}}\hat{\sigma}^2_{j_a}(m)\tau_{j_a} + \frac{\tau_{j_b} - \tau}{\tau_{j_b} - \tau_{j_a}}\hat{\sigma}^2_{j_b}(m)\tau_{j_b} \tag{5.26}$$

   in order to guarantee that the scenarios produced are arbitrage-free.

This interpolation is applied for the scenarios associated with all dates $t \in [1, N]$ in the lookback window.

5.4.3 Option P&L scenarios

Each P&L scenario is created with risk-factor scenarios from the same date *t* in the past, according to the following formula:

$$\begin{aligned}\widehat{\Delta^h P}^{t}_{t_0}(\tau, m) = \widehat{FX}^{t}_{t_0+h} P\left(\hat{F}^{t}_{t_0+h}(\tau - \delta_h), \hat{\sigma}^{t}_{t_0+h}(\tau - \delta_h, \widehat{m}) + \sigma^*(t_0, \tau, m) - \sigma(t_0, \tau, m)\right) \\ - FX(t_0) P\left(F(t_0, \tau), \sigma^*(t_0, \tau, m)\right)\end{aligned} \tag{5.27}$$

where,

$\hat{F}$ are the futures scenarios given in Eq. 5.7,

$\widehat{m}$ are the adjusted moneyness given $\hat{F}$,

σ^* is the implied volatility derived from the quoted market price,

σ is the fitted model implied volatility,

$\widehat{FX}$ and FX denote the scenarios and initial FX rate respectively.

Time-to-maturity and moneyness are consistently adjusted over the scenarios. Contrary to futures contracts, FX is applied for the whole option value, which is in keeping with the margining process, and accurately reflects the impact of FX exposure on the potential closeout cost in the event of a Clearing Participant default. Note that the implied volatility scenarios are corrected by the term $\sigma^*(t_0, \tau, m) - \sigma(t_0, \tau, m)$ which accounts for any potential initial mismatch between fitted and observed implied volatility (and prices).

Once the P&L scenario has been defined for a single option, it is possible to scale by the position quantity and the contract size to calculate the position-level P&L scenarios for this contract:

$$X_{i_{\tau,m}}^{t} = \widehat{\Delta^h P}_{t_0}^{t}(\tau, m) Q_{\mathrm{i}} S_{\mathrm{i}} \qquad \forall t \in [1, N] \tag{5.28}$$

where:

$X_{i_{\tau,m}}^{t}$ is the position-level P&L scenario (t) for option *i* with time-to-maturity τ and moneyness *m*

$\widehat{\Delta^h P}_{t_0}^{t}(\tau, m)$ is the single-contract-level P&L scenario (t) for option contract *i* with time-to-maturity τ and moneyness *m*

Q_i is the quantity of option *i*, in a given portfolio

S_i is the contract size of option *i,*

The position-level P&L scenarios can then be aggregated across all positions within the portfolio, along with those of the futures positions, (see Section 4.5) to calculate the overall portfolio-level P&L scenarios:

$$X_t = \sum_i X_i^t \qquad \forall t \in [1, N] \tag{5.29}$$

It is these portfolio-level P&L scenarios that are carried forward to the next stage of the FHS calculation (see Section 6.6).

6. Filtered Historical Simulation

The FHS methodology is based on the simulation of the distribution of future returns (scenarios) by sampling the realized past returns, scaled by the ratio of past and most recent volatility estimate. Compared to a historical simulation approach (HS), which directly identifies the scenarios with the past returns and thus assumes that returns are independent and identically distributed (i.i.d.), the FHS directly models the volatility dynamics, without making any parametric distributional assumptions. This point is a strong advantage as it allows to capture non-trivial statistical properties of real returns distributions, such as asymmetry (skewness), fat-tails (kurtosis), volatility clustering, and tail dependence. Finally, the FHS methodology is not limited to equity products and/or linear instruments and it will be here applied to futures and options.

The computation of initial margins within the chosen FHS-CVaR model extends the methodology applicable to cash equity products cleared by EuroCCP. In this section, we describe in detail the computation of IM within the FHS model.

6.1 Data inputs

The inputs for the scenario generation steps are the time-series of the relevant risk-factors. In the case of equity index derivatives, the relevant risk factors are:

- Futures relative logarithmic returns at pivot time-to-maturity points (see Section 4);
- Implied volatility absolute returns at pivot time-to-maturity and moneyness points (see Section 5); and
- FX spot rate relative logarithmic returns to account for potential mismatches between the risk factor time-series currency and the Clearing Participant margining currency.

6.2 Volatility and residuals estimation

One of the primary aims of the IM methodology is to capture the changes in volatility, and to use these to produce future return scenarios that are consistent with the present level of volatility[7]. To achieve this, the FHS incorporates a dynamic measure of market volatility.

The starting assumption of the FHS model is that the returns are well modelled as:

$$p_t^i = \hat{\sigma}_t^i \varepsilon_t^i \qquad (6.1)$$

where ε_t^i are normalised residuals (unit variance) and $\boldsymbol{\hat{\sigma}_t^i}$ is the forecasted instrument volatility on day t.

The volatilities are estimated from the historical returns of futures pivot points (Eq. 4.5) using an Exponentially Weighted Moving Average model (EWMA):

$$\left(\hat{\sigma}_t^i\right)^2 = \lambda\left(\hat{\sigma}_{t-1}^i\right)^2 + (1-\lambda)\left(p_{t-1}^i\right)^2 \quad , \qquad t \geq 2 \qquad (6.2)$$

The normalised residuals are then estimated as:

$$\hat{\varepsilon}_t^i = \frac{p_t^i}{\hat{\sigma}_{t+1}^i} \qquad (6.3)$$

Repeating the calculations (6.2) and (6.3) for all rows of the futures returns matrix (Eq. 4.5) gives the following matrix of normalised residuals:

[7] Only applicable to futures price returns.

$$\hat{\varepsilon} = \begin{pmatrix} \hat{\varepsilon}_t^1 & \hat{\varepsilon}_t^2 & \dots & \hat{\varepsilon}_t^M \\ \hat{\varepsilon}_{t-1}^1 & \hat{\varepsilon}_{t-1}^2 & \dots & \hat{\varepsilon}_{t-1}^M \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\varepsilon}_{t-N^*+1}^1 & \hat{\varepsilon}_{t-N^*+1}^2 & \cdots & \hat{\varepsilon}_{t-N^*+1}^M \end{pmatrix} \tag{6.4}$$

The estimation of the volatilities according to Eq. (6.2) requires the specification of a seed, the initial value $\hat{\sigma}_1^i$. Given the fully available return time series p_j with j = 1, …, *t*, the seed value σ_1^2 is computed as the average over the first N_0 squared returns and the values are propagated using recursion from there on. The selected value for N_0 is 200 days, which corresponds approximately to twice the characteristic period of the EWMA for λ = 0.99, and therefore reflects the timescale of the volatility process.

6.3 Filtered scenarios generation

Once determined, the normalised residuals can be used to generate a set of hypothetical *MPOR*-scaled (ie. *h*-day) return scenarios for a defined number of scenarios N (where *h* = 3 and N = 700).

To obtain *h*-day return scenarios, we sum 1-day residuals over *h* consecutive days in order to capture possible auto-correlations, ie. trending or mean-reverting behaviours. The sums of residuals are then multiplied by the last available volatility, which under the EWMA volatility process assumption, is the one-day ahead forecast volatility. Note that the summation across residuals requires the use of logarithmic returns.

The *MPOR*-scaled hypothetical return scenario for instrument *i* and scenario *k* is calculated as:

$$\hat{r}_k^i = \hat{\sigma}_{T+1}^i \sum_{j=0}^{h-1} \hat{\varepsilon}_{T-k+1-j}^i \tag{6.5}$$

In matrix notation, the *MPOR*-scaled hypothetical returns are represented as:

$$\hat{r} = \begin{pmatrix} \hat{r}_1^1 & \hat{r}_1^i & \dots & \hat{r}_1^I \\ \hat{r}_k^1 & \hat{r}_k^i & \dots & \hat{r}_k^I \\ \vdots & \vdots & \ddots & \vdots \\ \hat{r}_N^1 & \hat{r}_N^i & \cdots & \hat{r}_N^I \end{pmatrix} \tag{6.6}$$

Each column of (6.6) represents a set 700 scenarios of hypothetical 3-day returns for instrument *i*.

6.4 FX treatment

For products denominated in a currency different from the portfolio base currency (ie. EUR), simulations of the relevant currency pairs are also required. The latter can be obtained by modelling the foreign exchange (FX) spot rate return p_t^{fx} for currency pair *fx* as:

$$p_t^{fx} = \sigma_t^{fx} \varepsilon_t^{fx} \tag{6.7}$$

The *MPOR*-scaled hypothetical return scenario for currency pair *fx* and scenario *k* is then calculated in the same manner as described in Sections 6.2 and 6.3, with instrument *i* being replaced with currency pair *fx*. These fx return scenarios $(\hat{r}_t^{fx})$ are then applied to the corresponding current FX spot rate $FX(t_0)$ to generate a set of *N* FX scenarios:

$$\widehat{FX}_{t_0+h}^t = FX(t_0)\hat{r}_t^{fx} \tag{6.8}$$

The FX scenarios are then used, along with the corresponding futures and options return scenarios to calculate a set of P&L scenarios in the portfolio base currency. Eq. 4.10 and 5.27 illustrate how this is done for futures and options respectively.

6.5 P&L scenarios

6.5.1 Futures P&L scenarios

The process to generate contract-level P&L scenarios $\widehat{\Delta^h F_{t_0}^t}(\tau)$ for a future of time-to-maturity τ for all dates $t \in [1, N]$ of the lookback window is described in detail in Section 4.5.

The returns scenarios used to generate these contract-level P&L scenarios are calculated using the FHS process described in Sections 6.2 and 6.3.

6.5.2 Option P&L scenarios

The process to generate contract-level P&L scenarios $\widehat{\Delta^h P_{t_0}^t}(\tau, m)$ for an option of time-to-maturity τ and moneyness m for all dates $t \in [1, N]$ of the lookback window is described in detail in Section 5.4.

The inputs are return scenarios for the risk factors, namely futures returns at pivot time-to-maturity points and implied volatilities returns at pivot time-to-maturity and moneyness points. As described in Section 5.3.2, implied volatility scenarios are calculated based on absolute returns, rather than logarithmic, and are left unscaled in line with a pure HS methodology. The futures returns are calculated using FHS, as described in Sections 6.2 and 6.3, and are fully consistent with the calculation for futures P&L scenarios.

6.6 Margin calculation

6.6.1 Expected Shortfall risk measure

Margin requirements are obtained by calculating an ES risk measure for the portfolio based on the matrix of simulated P&L scenarios.

Given a random vector of portfolio P&L or return scenarios $\vec{x} = (x_1, \dots, x_N)$, the ES risk measure with confidence level α is estimated as follows[8]:

1. Sort $\vec{x}$ in ascending order, where $x_i \leq x_{i+1}$:

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} \quad (6.9)$$

2. Compute $N_\alpha = \lfloor N(1-\alpha) \rfloor$ where $\lfloor y \rfloor$ is the closest integer smaller or equal to y. For example, if $\alpha = 99\%$ then $N_\alpha = 7$ (for $N = 700$).

3. The ES (or conditional value-at-risk) is then estimated as the average of all scenarios up to, and including scenario N_α

$$ES_\alpha = \frac{1}{N_\alpha} \sum_{n=1}^{N_\alpha} x_n \quad (6.10)$$

[8] Note that flooring to the closest integer is more conservative than applying interpolation to estimate the empirical quantile.

6.6.2 Portfolio margin limit rule

The initial margin for a given portfolio is subject to a portfolio margin limit rule in accordance with Article 27(4) of the EMIR RTS.

Accordingly, the portfolio IM is calculated as a weighted sum of the gross and net portfolio margins:

$$IM^{FHS} = (1-c)IM^{FHS}_{gross} + c.IM^{FHS}_{net} \tag{6.11}$$

where $c = 0.8$, and IM is expressed as a negative value. Note that the additive property of the ES tail measure guarantees that $IM^{FHS}_{net} > IM^{FHS}_{gross}$.

The gross portfolio margin IM^{FHS}_{gross} is the sum of the individual instrument margins:

$$IM^{FHS}_{gross} = \sum_{i=1}^{I} f\,(w_i) \tag{6.12}$$

and the net portfolio margin IM^{FHS}_{net} is the margin of the netter portfolio including the effect of diversification:

$$IM^{FHS}_{gross} = f\left(\sum_{i=1}^{I} w_i\right) \tag{6.13}$$

where f denotes the ES function described in Section 6.3.

6.6.3 Derivatives netting rule

As an important particular case of netting, derivatives with the same underlying are considered as products for which full netting is allowed because their correlation stems by construction from a sufficiently fundamental and stable reason. As a result, all components of the portfolio that have the same underlying can be aggregated at scenario level as if they were one instrument and the aggregation described in the previous section applies to the netted scenarios across underliers.

7. Anti-Procyclicality Measure

Article 28 of the EMIR RTS sets out requirements for CCPs to limit the procyclicality of margin requirements to the extent that the soundness and financial security of the CCP is not adversely affected.

To comply with these regulatory requirements, EuroCCP applies an anti-procyclicality (APC) measure in the form of a stressed historical simulation (StressHS) margin which aims to reduce the procyclicality of the stand-alone FHS IM (ie. by applying the option set out under Article 28(1)(b) of the EMIR RTS).

The APC-compliant margin is then computed as the weighted sum of the FHS and the StressHS margins.

This section describes in detail the computation of the StressHS margin component and resultant APC-compliant margin.

7.1 StressHS margin

The StressHS margin component is based on a historical simulation, expected shortfall methodology. Simulation scenarios are obtained by combining a set of 650 of the recent past returns with a set of 50 returns that correspond to defined historical dates of exceptional market stress. The margins calculated using this model represent the potential expected loss for a given portfolio in stressed market conditions.

The incorporation of a less-variable margin component, derived from a relatively stable set of historical periods of stress provides for a conservative margin estimate that is less sensitive to the current dynamics of market volatility. As a result, margin requirements are more conservative during low volatility periods and less variable during high volatility periods.

7.1.1 Data inputs, scenario generation, and P&L scenarios

The StressHS computations are based on a combination of a set of historical stress scenarios with a set of the most recent scenarios, which together generate the set of simulated scenarios.

The 50 historical stress scenarios correspond to the EuroCCP stress-testing scenario dates, which are defined by EuroCCP as part of its overall stress testing framework. The selection and review of these stress scenarios is performed on a periodic basis and is subject to internal risk management and regulatory governance. Currently, EuroCCP's stress-testing scenario suite contains 50 historical stress scenarios covering all relevant markets that EuroCCP clears.

The 50 historical stress scenarios are complemented by the $(N - 50)$ most recent 3-day unscaled scenarios in the lookback period to form a total of N unscaled scenarios. The total length of the simulation window is $N = 700$ scenarios[9], and therefore the most recent 650 unscaled scenarios are included[10].

In accordance with ESMA regulatory guidelines, the StressHS margin component is based on unscaled (unfiltered) historical scenarios, and therefore the EWMA normalisation process described in Section 6 is not used for the generation of StressHS scenarios.

The processes described in Sections 4 and 5 can be used to directly generate a set of unscaled h-day P&L scenarios for the required number of recent lookback scenarios $N - 50$ (ie. 700 - 50 = 650) and for each stress period.

[9] The inclusion of 50 stress scenarios in the total simulation window of 700 scenarios is therefore sufficient to cover the tail of the distribution at the chosen confidence level (99%).

[10] Without duplication of scenarios, ie. where a scenario in the $(N - 50)$ lookback period corresponds with a stress scenario, it is treated as an invalid scenario for the purposes of scenario generation.

7.1.2 Margin calculation

Finally, the StressHS margin, IM^{SHS}, is computed similarly to Section 6.6, where the gross and net portfolio margin estimates are obtained by calculating ES risk measures for the portfolio based on the simulated P&L scenarios in matrix w^{SHS}, and the portfolio IM is calculated as a weighted sum of the gross and net portfolio margins in accordance with the portfolio margin limit rule.

Accordingly, the portfolio IM is calculated as a weighted sum of the gross and net portfolio margins:

$$IM^{SHS} = (1 - c)IM_{gross}^{SHS} + c.IM_{net}^{SHS} \tag{7.1}$$

where,

$$IM_{gross}^{SHS} = \sum_{i=1}^{I} f\,(w_i^{SHS}) \tag{7.2}$$

$$IM_{gross}^{SHS} = f\left(\sum_{i=1}^{I} w_i^{SHS}\right) \tag{7.3}$$

and where again c = 0.8, f denotes the ES function described in Section 6.6.1, the derivatives netting rule described in Section 6.6.3 applies, and IM is expressed as a negative value.

7.2 APC-compliant margin

Once the StressHS margin component has been calculated as per Eq. 7.1, it is combined with the FHS margin calculated as per Eq. 6.11 to produce the final APC-compliant margin in accordance with Article 28(1) of the EMIR RTS:

$$IM^{APC} = (1 - \eta)IM^{FHS} + \eta.IM^{SHS} \tag{7.4}$$

where η = 0.25.

8. Treatment of missing data

This section describes the treatment applied in instances of missing historical data. For derivatives, missing data can be partly recovered from existing one (e.g. through extrapolation and fitting) in addition to forward-filling.

8.1 Forward filling

In instances of a gap in risk factor data (futures and implied volatilities), the missing data points are forward-filled. This allows, as soon as a risk factor is available, that the following dates are all populated. This approach is deemed conservative as it creates returns over longer periods of time as long as the typical intervals of missing data do not exceed the length of the lookback window. After forward-filling the prices, returns are computed. Two situations then arise:

- incomplete term structure: returns are available for some maturities but data points surrounding the pivot maturities are not available, and hence require extrapolation; and/or
- fully missing term structure: a given date has no available maturities.

8.2 Incomplete term structure

8.2.1 Futures

The futures returns that serve as a basis for scenario generation are the pivot returns computed according to an interpolation/extrapolation scheme described in Section 4.3.3. Among these, the ones that stem from an extrapolation are flagged as "proxied". This flag is propagated when a risk factor return involves such a "proxied" pivot return in its own interpolation. Similarly, a risk-factor return that is extrapolated is flagged as "proxied".

8.2.2 Implied volatility pivots

First, the implied volatility pivots are obtained through a fitting process described in Section 5.3, as long as enough market data points are available for both time-to-maturity and moneyness dimensions. No implied volatility pivot points are therefore considered proxied. Second, the required data point is reconstructed according to a two-step process described in Section 5.4.2. Since the pivot points are expected to be well-chosen, no extrapolation is expected to happen at this stage.

8.2.3 Suppression mechanism for "proxied" data

At P&L aggregation time, all instruments that depend on proxied risk-factor returns are subject to "γ-suppression" which means they are afforded less netting benefit[11]. The reason for this is that the extrapolation operation spuriously reinforces the correlation up towards 1, which would drive the margin to zero for hedged long-short positions. The above protocol restores a non-zero margin with a level $(1-\gamma)/2$ times the gross margin. In this way, the suppression mechanism protects against margin underestimation arising from numerical approximations and proxying errors.

8.3 Fully missing term structure

8.3.1 Lookback dates

The case of a fully missing term structure is not expected to occur within the lookback window given that all available data is sourced directly from the exchange. If, however, an entire term structure was missing for one or more indices for a particular day[12], the futures prices and implied volatilities would be carried over from the

[11] This approach is identical to the methodology in place for cash equity products.

[12] This could arise, for example, in the event of a mis-match in holiday calendars between indices based in different countries.

previous day, effectively flattening all returns to zero, and the occurrence would be flagged for investigation by EuroCCP Risk Management.

8.3.2 Stress dates

Unlike with lookback dates, it is possible that the data for historical stress dates may be missing an equity index futures term structure in its entirety. Consequently, it is crucial that a conservative proxying approach is used for the stress dates, since the corresponding scenarios are used within the StressHS-CVaR margin component. For this reason, each historical stress date interval is assigned a fallback index, corresponding to the index that drove the selection of that date interval, and for which data is available[13]. Should the entire term structure of any index be missing in the stress interval, the return data from the fallback index is used over the whole term structure.

[13] Further details on the selection of historical stress date intervals are contained in the EuroCCP Stress-Testing Procedures.

9. Margin add-ons

In addition to underlying price volatility risk, EuroCCP is also exposed to other specific risks which require separate treatment in the form of margin add-ons. The following margin add-ons are applied by EuroCCP and included in the total initial margin requirement:

- Wrong-Way Risk (WWR) add-on
- Liquidity Risk (LR) add-on
- Large Position (LP) add-on

This section describes the methodology for calculating the above margin add-ons.

9.1 Wrong-way risk (WWR) add-on

The WWR add-on addresses the specific wrong-way risk arising when a Clearing Participant has a long exposure to its own stock or those of an institution belonging to the same financial group. In this case, the position attracts a 100% fixed margin, reflecting the fact that in case of default the position could be worthless.

In the case of index-based products, it is possible that a Clearing Participant's stock enters the composition of the index. In that case, their weight in the index α is used to evaluate the loss assuming that, in a default, the Clearing Participant's stock prices go to zero, leading to the following add-on:

$$WWR_i = -\max\left(\Delta(S,t_0)\alpha v_i + \frac{1}{2}\Gamma(S,t_0)(\alpha v_i)^2, 0\right) \tag{9.1}$$

where αv_i is the P&L of the underlying assuming the above, and where Δ and Γ are the aggregated deltas and gammas of the derivatives written on the underlying involving wrong-way risk. In theory, all the positions used to compute the initial margin should be reduced by α within the IM methodology but, for computational efficiency, this correction is neglected, making the overall approach slightly more conservative.

For a given Clearing Participant, the portfolio WWR add-on is the sum of all position WWR add-ons:

$$WWR_{portfolio} = \sum_{i=1}^{I} WWR_i \tag{9.2}$$

9.2 Liquidity Risk (LR) add-on

The LR add-on addresses the specific risk due to concentrated and/or illiquid positions in a Clearing Participant portfolio. This add-on captures the potential price impact of liquidating such positions in the event of a Clearing Participant default.

In such a case, liquidation of large and/or illiquid positions may incur additional costs beyond those captured in the standard margin computations, which address market risk only. The Liquidity Risk add-on described below is designed to capture the key risk factors on those portfolios, and to reflect the expected hedging and liquidation strategy that would be applied by EuroCCP in the event of a Clearing Participant default. Liquidation costs arising from offsetting positions across different maturity (date) buckets are treated accounted for within the vega liquidity risk component, and positions of contracts expiring within MPOR are accounted for separately within the vega liquidity risk component to limit the offsets provided by these contracts against other longer-dated maturities.

9.2.1 Futures

In order to compute the LR add-on, the following additional time-series for the first month futures contract are needed for each underlying *i*:

- the percentage bid-ask spread $Sp_{i,t}$, defined as:

$$Sp_{i,t} = \frac{Ask_{i,t} - Bid_{i,t}}{0.5\left(Ask_{i,t} + Bid_{i,t}\right)} \tag{9.3}$$

- the daily traded volume[14] denoted as $V_{i,t}$.

The LR add-on computed on day *t* for instrument *i*, as percentage of the instrument price, is defined as:

$$LR\%_{futures,i,t} = 0.5\overline{Sp}_{i,t} + \mu_i \sigma_{i,t+1} \sqrt{\frac{Q_i}{\bar{V}_{i,t}}} \tag{9.4}$$

where:

$\overline{Sp}_{i,t}$ is the simple average bid-ask spread computed over the last 250 observations of instrument *i*,
Q_i is the position size to be liquidated[15],
μ_i is a coefficient measuring market impact, with default value equal to 1,
$\bar{V}_{i,t}$ is the average traded volume computed over the last N_{liq} observations of V_t, and
$\sigma_{i,t+1}$ is the EWMA volatility computed on day *t*, as for instrument *i* (Eq. 6.2).

In absolute terms, the LR add-on for futures is given by:

$$LR_{futures,i,t} = v_{i,t} \times \left(LR\%_{futures,i,t}\right) \tag{9.5}$$

where $v_{i,t}$ is the last valid value of the position on instrument *i*.

The first part of the LR add-on accounts for the *exogenous* liquidity component, which does not depend on the position size and it covers half of the bid-ask spread to reflect the use of mid-prices in the margin computations.

The second part of the LR add-on accounts for the endogenous liquidity component, which considers the market impact of liquidating the position given the position size[16]. As the position to be liquidated gets larger, it will incur additional costs.

In cases where data inputs are missing:

- If an observation $SP_{i,t}$ (or $V_{i,t}$) is not available, the missing value is set equal to the last available rolling average $SP_{i,t-1}$ (or $V_{i,t-1}$).
- Where observations of $SP_{i,t}$ are not available at the start of the observation window, the spread time series is pre-filled with conservative default values, Sp_i^*, set to 5% in order to start the rolling average calculation (typical bid-ask quotes for large caps are in the range of few percentage basis points).

[14] Where required, the primary listing of the instrument is used to avoid underestimating the instrument liquidity.

[15] For each Clearing Participant and for each index, position size is determined aggregating the deltas of all positions on the index across all accounts, so long and short positions are allowed to cancel out.

[16] The shape of the market impact function has been widely investigated in the literature and the square-root form with γ_i = 1 adopted in Eq. 5.4 has been shown to perform well with respect to observable trading data. Commercial applications using the square-root form of the market impact include for instance Barra and Bloomberg.

- Similarly, where observations of $V_{i,t}$ are not available at the start of the observation window, the volume time series is pre-filled with conservative default values Q_i/π, where π = 0.2. This implies that on the first trading day, $Q_i/V_i = \pi$. In other words, it is assumed that any position in an IPO stock constitutes a fraction π of the unknown tradeable volume[17]. This assumption fades out progressively as actual volume data are observed. For new IPOs, the volumes over the first week of trading are discarded as these are not representative of the standard market trading activity.

For a given Clearing Participant, the LR add-on for futures contracts on day *t* is calculated as:

$$LR_{portfolio,futures,t} = \sum_{i=1}^{I} LR_{futures,i,t} \tag{9.6}$$

9.2.2 Options

The LR add-on for options is covered and modelled assuming that delta and vega are macro hedged before proceeding to an auction of the hedged portfolio. The LR add-on described above is then applied to the delta exposure in the underlying instrument (in the case of an index the most liquid futures will be used as hedging instrument). The vega add-on accounts for the residual risk which cannot be covered by trading in the underlying.

Delta Liquidity Risk Component

To compute the liquidity risk of the delta-hedge, let us consider the case of one option. Delta-hedging is done by short selling immediately and exactly Δ shares of the underlying *S* for each long unit of exposure to the option and vice versa. A portfolio is hedged with respect to all derivatives by using the aggregated, portfolio delta $\Delta(S(t),t)$. The newly-hedged portfolio is then liquidated over the *MPOR* and the delta-hedge covers exactly the variation of the options' prices due to the underlying price variation along this path, while the liquidity risk is “transferred" to the amount of underlying needed, hence the contribution of the portfolio delta:

$$LR_{option-delta,t} = \left|\Delta(S(t),t)\left(LR\%_{future,underlying,t}\right)S(t)\right| \tag{9.7}$$

where $LR\%_{future,underlying,t}$ is the LR add-on computed accordingly to the methodology for futures.

For delta exposure, full netting is allowed across derivatives on the same underlying due to the strong correlation observed across futures contracts.

Vega Liquidity Risk Component

The liquidity risk associated with the option price change due to volatility is covered by hedging the vega of the options portfolio, which is done, in practice, by executing ATM straddles at the appropriate maturity buckets. A predefined constant $\overline{\sigma_m'}$ is a conservative estimate of the cost of hedging one unit of vega for the *m*-th maturity bucket, so that the vega cost for the *m*-th maturity bucket is calculated as:

$$LR_{option-vega,t,m} = v_m(S(t),t)\overline{\sigma_m'} \tag{9.8}$$

where $v_m(S(t),t)$ is the aggregated vega for the *m*-th vega maturity bucket. In practice, $\overline{\sigma_m'}$ is calibrated on a regular basis from bid-ask spreads for ATM straddles, directly observed from closing bid and offer of the call and put that the straddle comprises. Although the implied volatilities observed at different maturities are generally highly correlated, a conservative estimation of the total vega hedging costs is calculated by taking the maximum of:

1. the total hedging cost for all long vega maturities, and
2. the total hedging cost for all short vega maturities.

[17] In practice, trades larger than 10% of the daily volume are rarely observed.

Very short dated vega (<1 week to maturity) is treated separately and, as a result, the add-on for these positions will be calculated independently of other maturities.

The overall the vega liquidity component is calculated as:

$$LR_{option-vega,t} = \max\left(\sum_{m=1}^{M} \max(LR_{option-vega,t,m}, 0), \sum_{m=1}^{M} \max(-LR_{option-vega,t,m}, 0)\right) \quad (9.9)$$

Total Options Liquidity Risk Add-On

The overall liquidity cost incurred is finally calculated as the sum of the liquidity cost of the delta-hedging and of the vega contribution:

$$LR_{portfolio,option,t} = LR_{option-delta,t} + LR_{option-vega,t} \quad (9.10)$$

The options liquidity add-on will be calculated for each index independently, with no hedging benefit recognised for offsetting short and long positions with different index underliers.

9.3 Large Position (LP) add-on

The LP add-on reflects the potential excess stress loss in a Clearing Participant portfolio over and above available financial resources. This add-on captures the excess loss exposure taking into account available resources in the equity derivatives segment of the mutualised Clearing Fund (under a Cover-2 standard) and the Clearing Participant's individual margin collateral posted.

For a given Clearing Participant, the portfolio LP add-on is calculated as follows:

$$LP_{portfolio} = min[0, MaxSTL - TM + \varsigma_i . DF] \quad (9.11)$$

where,

$MaxSTL$ is the maximum equity derivatives-related stress loss observed for the clearing member (expressed as a negative value).

TM is the total equity derivatives-related margin requirement including WWR for the Clearing Participant (expressed as a negative value).

DF is size of the equity derivatives segment of the EuroCCP Clearing Fund.

ς_i is defined by Clearing Participant, with a maximum (default) value of 0.45.

The LR add-on has been removed from the total margin requirement to ensure that this figure is comparable with the stress scenarios, which deal exclusively with market risk.

Appendix A – Margin model parameters

Table A.1 – FHS and StressHS margin model parameters

Parameter	Symbol	Value
Lookback window	N	700 days
EWMA decay factor	λ	0.99
EWMA seed period	-	200 days
Portfolio margin limit coefficient	c	0.8
StressHS margin coefficient	η	0.25